Space, Freedom, and Law: Maybe Space is *All* that Matters

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ABSTRACT

The traditional causal, *vectorial* approach to formulation of physical law is contrasted to the *analytic* approach which is based on *variational principles*. Reviewing the work of Toffoli, these two "paradigms" are considered in relation to the idea of *parsimony* arising from *prodigality*. The second law of thermodynamics and Darwinism are taken as further examples of global, non-causal principles similar to the variational principles, which may also be considered as cases of parsimony arising from prodigality, or law arising from freedom. *Space* is considered in a generalized, or abstract, sense, not as an arena for location of events and application of laws, but rather as that which affords freedom, and hence as that which may be both necessary and sufficient as the ultimate basis of physical laws—i.e. "all that matters". As a final remarkable example of constraint, or law, arising as an epiphenomenon of space, the variational-principle analogue of Bayerlein, Sharp, and Wheeler, from geometrodynamics, is mentioned, which is the basis of Barbour's theory that time itself arises as an illusory epiphenomenon from a space of all possibilities.

Keywords and phrases: freedom, law, variational principle, parsimony, prodigality, epiphenomenon

1 INTRODUCTION

Under the presumption that we all have a predilection to think spatially and visually, I invite the reader to consider the following. Mr. Feynman has said, in the introduction to his lectures on "The Character of Physical Law" (Feynman 1967), that science is concerned with the discovery and appreciation of the patterns and rhythms of Nature that may not be immediately aparent to the eye. The essence of the scientific mode of discovery and appreciation of the world is scientific analysis, which I think of as a kind of *transformation* of information, followed by *compression* of information into the form of "law". [1]

A transformation, in the sense in which I am using the word (after the mathematical sense), is just a change of the presumed coordinate system in information space, perhaps accompanied by magnification of certain aspects with respect to others. It is a generalized analogy to the simple rotational transformation effected when an animal cocks its head in an expression of puzzlement, perhaps as an attempt to view the scene in such a way that it becomes recognizable. Imagine that we might be able to pan around, and view Mr. Feynman's patterns and rhythms of Nature from different points of view in some sort of perception space.

Metaphorically speaking, perhaps what appeared as a disk from one point of view will appear much more linear from another point of view. So we seem to obtain quite different descriptions or explanations of reality, depending on our point of view. As a concrete analogy, imagine watching a point on the edge of a uniformly-rotating disk. Viewed from "above", the motion of the point is quite simply cyclic. Viewed "edgewise", the point oscillates, exhibiting a pattern of variation in position, velocity, acceleration, etc. that appears sinusoidal. So in a more abstract sense, after a transformation, taking a different point of view in the analysis of nature, perhaps what appeared as *rhythms* now appear as *patterns* (the effect of a Fourier transform), and so forth.

The salient point is that the essence of the correlations of the world remains *invariant*; only the point of view changes. This is significant insofar as even notions of causality and objectivity may turn out to be *covariant*, that

[1]The view of scientific law as information compression was propounded by Solomonoff (Solomonoff 1964).

is, dependent on point of view. The relation of causality can be seen as an essentially logical, rather than temporal, relation (Rosen 1995), and in fact it has already been suggested that what quantum mechanics is "trying to tell us" is that "correlations have reality; that which they correlate does not" (Mermin 1998).

1.1 General Notions of Space

Most of us operate within a scientific and philosophical tradition in which space is considered to be some sort of primordial arena in which things are located and to which external laws are applied in order to make things behave as they do. I would like to consider space more radically as that which affords *freedom* (possibilities) but which also accounts for *law* (constraint), as a consequence of freedom.

Geographic space affords degrees of freedom as well as constraint to ecological-evolutionary dynamics, and has been recognized to figure essentially in the patterns and rhythms ultimately observed. I wish to push the issue a bit further and entertain the intriguing possibility that space, understood in the broadest sense, may be the very reason for change, both in terms of freedom as to possibilities and in terms of constraint as to specific direction.

In the introduction to a work on the foundations of logic, in which the primordial notion was that of *distinction*—i.e. dintinguishability, or difference—George Spencer-Brown made the metaphysical remark that "when a space is cloven or taken apart, a universe comes into being" (Spencer-Brown 1969). Following this severely parsimonious bent, the present essay is motivated by the wish to discover how much of physics (or biology, or whatever) might be just logic. Perhaps ironically, the thesis is that parsimony, the apparently simple laws that describe the rhythms and patterns of nature, actually depends on prodigality, or the provision of a rich space of possibilities.

The dynamics of Newton assumed space and time as absolute frameworks for the placement of phenomena; the philosophy of Kant assumed that space and time were constructs that necessarily preceded thought. Leibniz and Mach, in contrast, tended to regard space as something subsumed in the integration of connectivity and correlations between phenomena. Either way, space provides a way of understanding both possibilities and impossibilities, or, passing from the qualitative to the quantitative, a way of understanding degrees of likelihood, or probability. That probability is of the essence in theoretical physics has been recognized; Schrödinger said that "physical laws rest on atomic statistics and are therefore only approximate" (Schrödinger 1944); Anandan said that "there are no fundamental causal laws, but only probabilities for physical processes" (Anandan 2003).

1.2 Patterns in Space

When we ask *why* we observe the patterns that we do, or *how* a process works, we generally expect causal explanations that specify the particular constraints that force the observed outcomes. We seek prior conditions and rules for temporal consequences.

Nevertheless, occasionally it is noticed that things appear to have the uncanny tendency to behave of their own accord. We see various sorts of *epiphenomena* in the world—patterns and laws that seem to emerge collectively, perhaps statistically, perhaps by feedback loops of self-reference (Hofstadter 2007). And sometimes either we, or nature, seem to discover "shortcuts" to prediction or outcome. I refer to such "tricks" as the principle of least action, or the maximum entropy production principle, and to nature's apparent ability to "solve" certain optimization "problems" (such as the formation of a saddle-shaped soap film) at a stroke, without working very hard at it.

Not to detract from the magic of nature, but I think the impression of a shortcut is a bit illusory, based partly on our preference for causal-sequential reasoning, and partly on our ignorance of the unexpected circumstance that prodigality may underlie parsimony.

The word "parsimony" is used here (after (Toffoli 2003); see also (Hildebrandt & Tromba 1996)) to refer to a certain economy of form or recurrence of method as observed in Nature. The parsimony of Nature is exhibited in various minimal- and optimal principles, and more generally in the existence of simple "laws" that describe her habits.

The word "prodigality" is used here to refer to a general notion of expansion in an abstract space of possibilities (like phase space). The general sense is of the profligate aspect of Nature, as she goes about her business of abhorring vacuums and generally filling the bill that "what can happen, will happen".

Perhaps it is careless to mix discussion of abstract space with discussion of "real" space, but I am trying to follow the "geometrodynamics" (Misner, Thorne & Wheeler 1973) and "it from bit" (Wheeler 1990) traditions by allowing that physics may best be understood in terms of space, and space may best be understood in terms of information. Geometrodynamics is the program, primarily after Einstein's relativity theories, to understand fundamental physical phenomena purely in terms of geometry. "It from bit" (physicist John Wheeler's catch-phrase) is the program to understand the possible underlying information-theoretic basis of all physical phenomena.

Here is the agenda: Einstein has been quoted: “What really interests me is whether God had any choice in the creation of the world” (Misner et al. 1973). It may be that, with a change of perspective, and a concomitant change of what we expect of an explanation (in light of the fact that causality may be, loosely speaking, covariant rather than invariant), we might instead occupy ourselves with the inverse question of whether God has anything *but* choice in the creation of the world. *Space* represents that freedom of choice.

2 BACKGROUND AND DISCUSSION

The balance of this paper is a general discussion of the idea of *law* emerging as an epiphenomenon of *freedom*, or expansion in the space of possibilities.

2.1 Prodigality and Parsimony—Chickens and Eggs

Nature seems to exhibit, or perhaps even to *be* in essence, a surprising juxtaposition of prodigality and parsimony. That such incomprehensibly rich variety of phenomena as we see all around us could come with such apparent efficiency and economy of principle is a circumstance of such intrigue that it occupies our perpetual scientific fascination. [2] One recalls that the most significant connections are those found to exist between pairs, such as *chaos* vs. *self-organization*, or *disorder* vs. *order*, previously thought to be anathema. By employing concepts of space on different levels—locational and abstract—I will sketch here how I think these two contrasting aspects of nature—prodigality and parsimony— are not merely compatible, but somehow related.

It is in fact a chicken-and-egg question, I think. In trying to explain which came first, it is usual to explain how prodigality could come from parsimony, because simplicity is supposed to be beautiful, and is assumed to take priority in explanation. Not only are we strangely attracted to *minimal principles*, which describe Nature’s parsimony, but we would like to have a *minimum of principles* as well, and we would like them to be about minimal elements, yet they should have marvelous and far-reaching consequences. “Back to basics!” political candidates and religious fundamentalists beguile us, trying to take advantage of our simple-mindedness. We are most impressed, for example, when the simple egg of a cellular automaton can produce an unexpectedly rich variety of pattern, or perhaps even function as a universal turing machine (Wolfram 2002). But I think that the chicken came first, and I will suggest why I think that the evidence is that, at least sometimes, parsimony arises from prodigality, that law arises as an epiphenomenon of freedom.

2.2 Variational Principles and Analytic Mechanics

At the same time that Newton was formulating his laws of mechanics to summarize how things move through space, with a sort of temporal logic, certain *variational principles* were known which seemed rather mysteriously to summarize entire paths in space, but without providing the sort of causal explanation that was perhaps sought even at a time when scientists were more sympathetic to teleological explanation than they are now. The Newtonian approach is sometimes called *vectorial mechanics* since it takes force and momentum vectors as elemental quantities.

The term *variational principle* refers to small variations from a solution, or chosen path of nature, resulting in departures from the minimal or optimal value of some global scalar quantity characterizing the entire solution or path. The “calculus of variations” is the tool of mathematical analysis used with variational principles, for example to solve the problem of what form of curve a hanging chain will take (a catenary), or what path will take a rolling ball most quickly from one point to another (a cycloid). Instead of the ubiquitous differential equations that describe locally and immediately how one condition leads to the adjacent or next condition, variational principles describe the integral forms taken by entire solutions to problems.

Recognizing the mathematical equivalence of the variational and the vectorial approaches to mechanics, Newton’s contemporary Leibniz, followed by Euler and Lagrange, founded what is sometimes called *analytic mechanics*, since it employs methods of mathematical analysis involving limits, taking the scalar quantities *energy* and *work function* as elemental, instead of the vector quantities *force* and *momentum* of Newtonian mechanics. Analytic mechanics is at least as powerful as vectorial mechanics by the criteria of being able to predict new observations and being able to describe old observations with logical economy (Lanczos 1970).

I regard the passage from vectorial mechanics to analytic mechanics as something of a *transformation*, in the sense introduced earlier in this paper. Superficially, at least, vectorial mechanics seems more appropriate for providing causal explanation, insofar as it differentiates a system into causally-related parts, while analytic mechanics seems instead to provide something more akin to teleological explanation, insofar as it integrates the final form of a system as an auto-correlated whole, as if to reveal “aim”.

[2] It could be said that we are most fortunate, at least for our intellectual entertainment if not for our thermodynamic existence, to live in a world with so much opportunity for information compression.

The parsimony of nature is expressed in a number of variational principles, some examples of which follow. But as will be seen, this parsimony may actually have its roots in prodigality.

2.3 Some Variational Principles

Fermat's principle is an example of a variational principle of geometric optics, stating that a ray of light between two points takes the quickest route—that is, the path of least time. This is true even if the light passes through a medium of non-constant index of refraction, so that the ray is bent. For it will be bent in such a way as to go further in the medium of lower index (i.e. higher speed of propagation) and less distance in the medium of higher index (i.e. lower speed of propagation), so as to save time. You can't help wondering, in the tradition in which we have been taught to think, *how does it know*? Meanwhile, the wave-front description, evoking the image of a row of little marching soldiers who are retarded as they encounter slower going, may seem like a better causal "explanation" of why the light ray takes the path that it does, but these are, after all, just different ways of looking at the same thing.

Here the wave-front description exemplifies the vectorial approach. But it is interesting to note that the parsimonious solution (of least travel time) emerges as the collective result of any number of wave-front components radiating outward freely (subject only to the speed-of-propagation constraint of the different media), summing and cancelling among themselves. No one told the group of soldiers that they should get from "point A" to "point B" as quickly as possible, but this result emerged from their interaction.

Fermat's principle is actually a special case of the variational principle of *least action*. The principle of least action states that, given starting and ending points of a particle's trajectory, the path actually taken by the particle minimizes a rather mysterious and ambiguously-defined work function known as the action, which in this case is the time-integral of the Lagrangian, the difference between the kinetic and potential energies of the particle.

Richard Feynman, who explicitly appreciated the pleasure of recognizing old things from a new point of view (Feynman 1948) investigated the principle of least action in great depth, in a path of exposition beginning with his 1942 Ph.D. thesis on "The principle of least action in quantum mechanics" and continuing to a 1965 work on *Quantum Mechanics and Path Integrals* (Feynman & Hibbs 1965) in which the principle of least action was "explained" as a consequence of quantum mechanics, via the summation of the phase components of the complex-valued *probability amplitudes* for all possible quantum paths between two points.

According to quantum mechanics, a dynamic history, such as the path of a particle, is described in terms of probability amplitudes between observations; only when an observation is made, is an actual probability for measurement realized. Even this, it should be noted, does not represent a unique and definite value, but rather the peak of a probability distribution of a multiplicity of possible values. But on a deeper level, Feynman's work shows that even the well-defined statistical peak of the path of least action comes from summing a figure (probability amplitude) for *all* possible paths, allowing that most of these figures cancel one another (roughly analogous to the wave-front components mentioned previously). Insofar as no possible paths are ruled out *a priori*, one can see that a *minimal* principle oddly emerges as a statistical artifact of a *maximal* principle; *parsimony* in effect appears as an epiphenomenon of *prodigality* (Toffoli 2003).

Newton's concept of inertia is, in a sense, a variational principle: that a body, unaffected by any other constraints, should continue motion *in a straight line*. In effect, it will follow the shortest path between "point A" and "point B". Why should it exhibit such parsimony? Why not wander around a bit along the way? Toffoli, in posing the question of "how much of physics is *just* computation", argues in effect that the shortest path between two points, the "straight line", can be constructed as the statistical expectation when considering *all* possible paths (Toffoli 1998). His argument proceeds by the strategy of trying to make the "safest" guess as to the midpoint, given a start point and an endpoint, but no constraints (except speed-of-light) on motion. The peak of the distribution of possibilities for the midpoint is the point midway on a straight line between the two points. By then similarly finding the most likely quarter-points, and so forth, the straight line of inertial motion simply emerges statistically as the most likely path, if you don't know anything about Newtonian dynamics and wish to consider all possible paths.

It kind of makes you wonder whether Nature knows about Newtonian dynamics either. Maybe nature is not as parsimonious and decisive as we thought. Maybe the apparently clear-cut things that we observe are merely statistical peaks, the result of the fact that what we are most likely to observe is whatever is most highly-represented.

2.4 Second Law of Thermodynamics—Physics from Logic

As an eminent example of necessity that may be an epiphenomenon of chance, consider the second law of thermodynamics. Briefly, it originally described the non-conservation of available energy, the fact that temperature differences could do work and that work could make temperature differences, but that you would always lose ground going back and forth. In effect, what one expects to observe, in the large, is constrained tidily by the very statistics of how the configuration, in the small, is *not* constrained. For example, an equilibrated temperature may be an expected result of unconstrained diffusion of heat. The second law could be said to fit in the category of variational principles insofar as it is not a causal law, but describes global tendencies. Although it does not specify the minimization or optimization of a quantity (such as entropy), some of its corollaries, in the field of non-equilibrium thermodynamics, do.

From an information-theoretic point of view, the second law of thermodynamics concerns *distributions* and their *entropy*. The entropy of a distribution is a measure of its spread. If you plot all possible values against the probabilities for those values, so that the sum of all probabilities (the area under the curve) is equal to 1, the entropy of the distribution quantifies how close the distribution is to being flat, with height = 1/width. This measure happens also to be the measure of how much you learn, when you find out which of the possibilities actually is the case.

Viewing the world in terms of equivalence classes, which is a prerequisite for doing science at all (Rosen 1995), we see that the second law essentially states that less probable (i.e. less highly-populated) equivalence classes lead to more probable (i.e more highly-populated) equivalence classes: in a (phase) space of possibilities, one expects any given system to follow a path from smaller-volume equivalence classes to larger-volume equivalence classes, simply because there are more ways for that to happen.

The second law of thermodynamics, with its apparent connections to irreversibility or time asymmetry, is a most remarkable law, since it seems to be nothing but a rule of logical inference at heart (Jaynes 1957); an intangible Shiva, it ensures dissolution, as recognized by Clausius and Wiener, while yet providing the means of regeneration by the dynamics of non-equilibrium thermodynamics, as explored extensively by Prigogine (see, eg., (Kondepudi & Prigogine 1998), and via the reciprocal relations, investigated notably by Onsager (Onsager 1931).

The second law of thermodynamics stands as an example of constraint, in one aspect, arising from freedom, in another aspect, in both of two parallel interpretations of the second law, with reference to *physical entropy* (Denbigh & Denbigh 1985, Zurek 1989) and to *information-theoretic entropy* (Shannon 1948, Jaynes 1957). First, in the study of non-equilibrium thermodynamics, one sees in the *dissipative structures* of Prigogine (e.g. (Kondepudi & Prigogine 1998)) that freedom of flow in the presence of a gradient engenders differentiation and stability of form; [3] second, in the probabilistic inference of Jaynes (Jaynes 1957, Jaynes 2003) one arrives at the proper constraint as to what is to be expected, not by being pushed by physical considerations, but rather by being drawn, so to speak, by the methodical consideration of all possible outcomes (as exemplified by the derivation of the inertial path as "most likely").

2.5 Newtonian Dynamics from the Space of Possibilities

Historically, thanks in large part to the expository efforts of physicist Ed Jaynes (Jaynes 1957, Jaynes 2003), the second law of thermodynamics has come to be understood by many as the emergent effect of lack of constraint on what is known about a quantity yet to be observed.

In this school of thought, the *entropic dynamics* program (Caticha 1998, Caticha 2001) aims to derive the laws of mechanics from principles of inference alone, by providing for all possibilities within reason, yet postulating no a-priori physical constraints at all.

Toward this aim, and from our point of view of law arising as an epiphenomenon of freedom, what could be assumed as the most primitive measure of freedom, with which we might construct an informational space of possibilities? *Difference*, or *distinguishability* appears to be the most fundamental notion, and might serve to define the most primitive measure imaginable. Caticha constructs such a space, and perhaps it should come as no surprise that the same measure of distinguishability serves to define temporal separation and spatial separation (change being difference in time). The metric of the space is information distance, which can be well-defined using concepts of algorithmic information theory (Bennett, Gács, Vitányi & Zurek 1998).

3 WHAT DOES THIS HAVE TO DO WITH LIFE?

"*Evolution: it's not just a good idea, **it's the law***.: This no-nonsense interpretation of the seriousness of Darwinism plays nicely on the question of what the fundamental reasons for evolutionary change might be. Are biological or physical evolution essentially characterized by creative opportunism, or are they essentially mandated by law? Are they expressions of chance, or of necessity? [4]

[3] As in the familiar saying, "The flow of energy through a system tends to organize that system", attributed to R. Buckminster Fuller.

[4] cf. Monod's *Chance and Necessity* (Monod 1970).

If evolution is compulsory, how did it come to be legislated by nature? Evoking a natural philosophy of competitive determinism that is the bastard child of an unholy coupling of science to socio-economic rationale, we could accept that the evolutionary imperative is simply forced by the invisible fist of selfishness. [5] More radically, perhaps we will discover that, as in mechanics, so in biology: constraint arises from freedom, that the necessity of evolution is an epiphenomenon of chance. [6]

3.1 Darwinism as a Tautological Principle

Darwinism is commonly understood as "survival of the fittest". What does that mean? that whatever persists in time and spreads in space will be observed? But that's true of anything, right? It is just logic; in fact it is tautological! [7]

The statement is nonetheless surprisingly powerful, and akin to the second law of thermodynamics. Perhaps some biologists, when they perceive the tautology of the statement, balk, because it contains no biological causal explanation. If *fitness* is understood to be a collective measure with respect to one's environment, of likelihood of widespread and enduring occurrence, rather than an individual measure of self-dissemination and power to dominate potential competition, then it seems to be a concept not confined to biology, and perhaps even related to the "robustness" of entropic distributions (cf.(Demetrius & Manke 2004)). As such it is another example of apparent constraint of form being an emergent effect of the underlying freedom of variation.

In defense of general principles which seem to lack substance but which might be milked for more *laws* that emerge from the *freedom* of possibilities, one might point out that mathematics has great power, and it is tautological. When we discover that the same equations appear in different fields of study, we marvel at the economy of natural architecture, and remark that "God must be a mathematician." What nonsensical self-projection, to think that, because we can well describe the world in our language of choice, that the world must speak that language! Instead we should recognize that these phenomena which we are able to describe using similar statements in our language may have more in common than the description; they may be different manifestations of the same underlying principle.

3.2 A Digression into the Matter of Explanation

Did Feynman succeed in explaining the principle of least action in terms of quantum path integrals? I believe that even Feynman would say that he had not explained the principle of least action, but had merely succeeded in presenting it in a new way. Toffoli, whose work I have also referenced elsewhere in this paper, would like to "explain" the principle of least action without recourse to quantum mechanics, perhaps attaining a combinatorial view of it [8] (as Boltzmann offered a combinatorial view of entropy (Boltzmann 1886)), as another of nature's apparently parsimonious *minimum* extremal principles that in fact emerges as a statistical epiphenomenon of nature's prodigality (Toffoli 2003).

Since variational principles, like extremal- or optimal principles such as that of Maximum Entropy Production or the Le Chatelier–Braun Principle or the second law of thermodynamics, economically summarize expectations as well as observations, and have predictive power, yet appear to be in need of explanation, it seems appropriate to ask: what are we after, in an explanation? Most likely we seek to consolidate analogies (Hofstadter 2002), to relate new observations (and beliefs) to old observations (and beliefs). In that case, the business of science is more than description of the patterns and rhythms of nature by the discovery or invention of "laws" that most efficiently compress the information of observations and potential predictions (as proposed in Solomonoff's algorithmic information-theoretic "formal theory of scientific inference" (Solomonoff 1964)); science must in addition remain tethered to current modes of understanding, and must be obligated to proceed more or less continuously from the present model of the world to the next acceptable model of the world.

3.3 Life as a Manifestation of the 2nd Law of Thermodynamics

It has been suggested that the evolution of life on Earth may be a manifestation of the second law of thermodynamics (Schneider & Kay 1994), or that at least the second law accounts for much of its dynamics (Hamilton 1977, Wicken 1980, Brooks & Wiley 1988). It is fairly obvious that life requires "negentropy" (that is, energy or matter of low entropy, which can be degraded for purposes of growth and reproduction

[5]Allusion to Adam Smith's "invisible hand" which is supposed to steady capitalism on its boldly determined course, and also to Richard Dawkins' "selfish gene".

[6]But I would prefer the term *possibility*.

[7]Cf. (Lotka 1922)

[8]"Combinatorial" referring to counting: how many ways can one equivalence class appear, vs. how many ways can alternatives appear.

(Schrödinger 1944). In fact, life is like a refrigerator, depending on greater differences than it is ever able to produce.

What is less obvious is that the presence of life on Earth exhibits expected overall thermodynamic effects (Lovelock 1987), just as, for example, the atmosphere as a whole exhibits expected overall thermodynamic effects, such as enhanced degradation of energy gradients (Maximum Entropy Production) (Paltridge 1979, Kleidon & Lorenz 2005). One might venture to say that life arises as the mechanism to bring about these thermodynamic effects. But this view is generally not well-received because it is thought to smack of teleology (like the variational principles), instead of providing a causal (vectorial) explanation. Nevertheless, consideration of thermodynamics, along with combinatorial principles of self-organization, lead to the more widely-held view that life and its evolution are to be expected (Kauffman 1995), like hurricanes after summer's heat, given a big difference and a little logic.

4 SPACE IN RELATION TO TIME

The above-mentioned speculations are based on concepts from non-equilibrium thermodynamics, including the concept of irreversible processes and the concept of time asymmetry. We come around to such exotic concepts because the question of how space relates, not just to time, but to time asymmetry, is as yet unsettled and, I think, central to any account of dynamics.

The second law of thermodynamics has been "explained" by postulating an early cosmological condition of "flat" space, or smooth gravitational field (Penrose 1989), insofar as that is an extraordinarily improbable condition of space, one that might be expected to precede less probable (yet remarkable) conditions such as the universe as we know it. [9]

Such flat space, or distributed gravitational potential, is not easy to explain. Davies thought it might be the result of the *expansion* of space (Davies 1983), yet Page has argued that Davies' reasoning depends, in effect, on second-law presumptions (Page 1984)!

4.1 The Asymmetry of Time

The "arrow of time" (after (Eddington 1928)) refers to the apparent directionality of evolution, that is, *irreversibility* of the collective sequences of events, such as the "break" in a game of pool, as contrasted to *reversibility* of specific idealized sequences of events, such as individual billiard-ball collisions: a film of a break run backwards would appear absurd, while a film of individual collisions run backward would appear reasonable. There is apparently an essential difference between the evolution of an *ensemble* of events, versus the evolution of an individual sequence of events, inasmuch as the laws of physics, as expressed in equations in which time appears in the second power, are invariant with respect to a change of sign of time, and so are said to be time-symmetric. But in accordance with the 2nd law of thermodynamics, the evolution of collections, or distributions, obey statistical laws, agreeing with each other and with the arrow of time: the 2nd law essentially states that we expect differentiated distributions to become equilibrated.

While some have suggested that the asymmetry of time comprises nothing more than the effect of the 2nd law of thermodynamics (Boltzmann, for example), and that the sense of time would be reversed wherever the entropic trend were reversed, others have maintained that the forward direction of time is more generally related to expansion, rather than just to equilibration of distributions (Eddington 1928, Popper 1965, Denbigh 1989, Zeh 1992). This is a concept of expansion not just in ordinary space, but in state space, or the space of possibilities. In fact, in such a space it is seen that the volume of equilibrated distributions is larger than the volume of differentiated distributions, so that in a sense equilibration is expansion.

Irreversibility, or the asymmetry of time, has appeared as a bit of a mystery, since it has not been explained satisfactorily by causal arguments on the basis of known laws of dynamics which appear to be time-symmetric. But perhaps this is like trying to explain Darwin's principle on the basis of biology, when it is really pre-biology. Perhaps irreversibility (whether it is considered as "expansion" in a generalized space or otherwise) should be regarded as pre-physical—if not just logic, then at least akin to logic, as logical consequence parallels temporal consequence. Indeed, some physicists have turned the tables and asserted that time asymmetry is a precondition of human experience (Görnitz, Ruhnau & Weizsäcker 1992), and it is the symmetry of the fundamental equations of dynamics that is in need of explanation!

[9] Vanishing *Weyl* tensor of space-time curvature represents maximum overall gravitational potential, and such phenomena as stars and biospheres can reasonably be expected to follow.

4.2 Space, or Freedom, as the Prime Mover

Could it be that all dynamics are driven by the metaphorical expansion of phase space in the cylinders of the entropy engine of *irreversible* processes? Prigogine (who, incidentally, strove to explain irreversibility on the basis of time-asymmetric physical law) remarked that irreversibility is the *flow of correlations* (Prigogine 2003): objects or regions that were once entirely differentiated and separate meet and interact, and though they mix and become homogenized, microscopic correlations spatially disperse, but thenceforth retain the information that would theoretically allow retrodiction of a past, more "ordered" or low-entropy state. [10] Is the dynamo of creation driven by the flow of correlations, from an unlikely state of separation to a likely state of commingling, from past constraint to future freedom?

It appears that initial spatial order—constrained *state*—together with freedom of dispersion in space (either ordinary space or state space) accounts for temporal order—constrained *dynamics*—which in turn lead to subsequent constrained states ("dissipative structures"), and so on; in this way spatial- and temporal order self-propagate, given freedom, or "space".

Law, as Toffoli uses the word (Toffoli 1998), is the specification of *path*, as contrasted to (or complemented by) specification of *state* (which might simply be called *description*). In this sense, scientific law is perhaps concerned primarily with Nature's "rhythms" and only consequently with her "patterns". Toffoli recognized that the multiplicity of ways of getting from "here" to "there" is the dynamic entropy analogue to the familiar static entropy quantified by the multiplicity of ways that a system can be in a state belonging to a given equivalence class of states. His insight is analogous to that of Dewar, who offered an information-theoretic explanation of the Maximum Entropy Production principle in terms of *paths*, in a way analogous to familiar entropic arguments concerning *states* (Dewar 2005). The upshot is that state and path appear as "duals". This is reminiscent of the "dual" relationship between electric and magnetic fields, the variation of one accounting for the other, giving rise to self-propagation.

4.3 Geometrodynamics; Space as an Effect of Relations

As mentioned previously, there are two fundamentally different views of space: either it is an arena for events, a sort of empty frame of reference, or else it is the sum of relations between things. The latter view facilitates the realization that coordinate systems, or frames of reference, provide a way of specifying locations in space, but they need not be taken to be absolute or be identified with the space. Spatial coordinates are *covariant* under transformation—change of coordinate system—but not *invariant*, or of any absolute significance.

Special relativity came from the assumption that things should look the same in any uniformly-moving frame of reference, that there was no privileged frame of absolute rest. Taking the speed of light as invariant—appearing the same regardless of frame of reference—had the counter-intuitive implication that time, as well as position, must be covariant—dependent upon frame of reference. Hence space and time as independent entities were subsumed in the relations of space-time.

General relativity came from the assumption that things should look the same in *any* frame of reference, whether accelerated or subject to gravitation. Again, a modified view of space (space-time) itself was used as a way to understand relations—in particular gravitational interaction could be understood as curvature of space-time. Hence space, time, and matter as independent entities were subsumed in the relations of curved space-time (Lanczos 1970). [11]

The most remarkable aspect of relativity, not widely appreciated, I think, is the extent to which it explained physics as geometry. It is so amazing, one is challenged to distinguish physics from logic. Even Einstein believed, for a long time, that "general covariance" was a deeply physical principle, until the mathematician Kretschmann finally convinced him that it, also, had no physical significance (Barbour 2000).

4.4 Time as an Epiphenomenon of Space

The *geometrodynamics* of Einstein, Wheeler, Barbour, and others [12] is at heart a model for deriving physical laws from properties of space. Geometrodynamics concerns in particular the idea of *geodesics*—shortest paths between points—in space more and more difficult to imagine.

In a 1962 paper, Baierlein, Sharp, and Wheeler presented an analogue of a variational principle for general relativity (Baierlein, Sharp & Wheeler 1962), in which temporal information appears to be contained in purely

[10] This rather surprising fact, that the so-called *Gibbs entropy* remains constant through an irreversible process (Gull 1989), is the basis for the assertion that "entropy is an anthropomorphic concept" (Jaynes 1965).

[11] And in a sort of figure-ground switch of point of view, one could view matter as being a manifestation of fields, instead of fields being manifestations of matter.

[12] Including eminently Arnowitt, Deser, and Misner.

spatial specifications, given two geometries of space (analogous to the beginning and ending points from which a path may be determined by the principle of least action).

Barbour has explored this further, and concludes that, in effect, time has no physical significance, but emerges as a sort of statistical epiphenomenon, albeit illusory, from a configuration space that includes all configurations (Barbour 2000). In a way at least roughly analogous to Feynman's derivation of the path of least action by integration of all paths in a region, and to Toffoli's derivation of the inertial straight-line path as a sort of peak of a statistical distribution, Barbour derives conventional paths of evolution in time (from configuration to configuration in a timeless space called "Platonia") as geodesics, employing the generalized variational principle. Once again, space yields law; the chicken of prodigality lays the golden egg of parsimony.

5 SUMMARY

Whereas space is commonly thought of as an empty arena in which its contents evolve according to law, we've become acquainted with some examples showing that perhaps space, understood broadly, accounts for law. The variational principles, the second law of thermodynamics, and Darwinism turn out to have in common some purely logical, or combinatorial, aspects, which lead to the view that Nature's apparent parsimony, economy, and law, may actually be epiphenomena that emerge from her prodigality, multiplicity, and freedom.

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